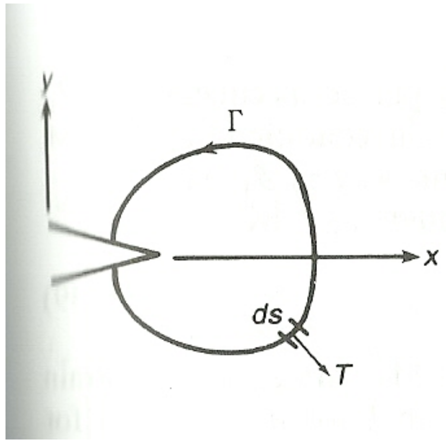
J Integral



Γ Contour drawn around a crack tip to define *J* integral

- *J* Integral provides fracture mechanics approach for lower strength ductile materials (both elastic and plastic deformation)
- Rice has solved two-dimensional crack problems in the presence of plastic deformation
- The line integral related to the energy in the vicinity of a crack is given by

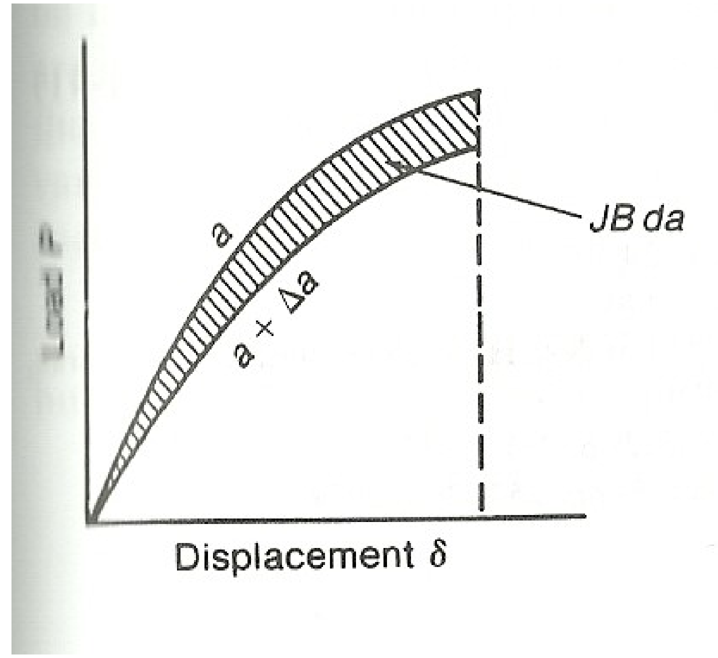
$$J = \int_{\Gamma} \left(W\,dy - T\frac{\partial u}{\partial x}\,ds \right)$$

Where $W = \int \sigma_{ij}\, d\varepsilon_{ij}$ is the strain energy per unit volume due to loading

Γ= path of the integral which encloses the crack
T = outward traction (stress) vector acting on the contour around the crack
u = displacement vector in x-direction
ds = increment of the contour path
x, *y* are the rectangular coordinates
$T(\partial u/\partial x)\,ds$ = rate of work input from the stress field into the area enclosed by Γ

Fracture occurs when the J integral reaches a critical value. J has units of N/m

- Rice has shown that the J integral is path- independent, i.e., J-integral = 0 for closed contour.
- *J* can be determined from stress analysis by FEA around the contour enclosing the crack.
- *J* integral can be interpreted as the potential energy difference between two identically loaded specimens having slightly different crack lengths.



Interpretation of J integral, B is specimen thickness

$$J = \frac{\partial u}{\partial a} = G = \frac{K^2}{E'}$$

Where E’ = E (plane stress)

E’ = E/(1-v^2) (plane strain)

The value of J (obtained under elastic-plastic conditions) is numerically equal to the strain-energy release rate (obtained under elastic conditions).

- The change in strain energy *ΔU* associated with an incremental crack advance Δa is shown in figure and equal to *JΔa* or *J* = (*dU*/*da*), where

$$J = \int_0^p \left(\frac{\partial \delta}{\partial a} \right)_p dP$$

$$J = -\int_0^\delta \left(\frac{\partial P}{\partial a} \right)_\delta d\delta$$

- *J* is the energy made available at the crack tip per unit crack extension
- *J* is also known as crack driving force

- *J* is the measure of lowering of potential energy during crack growth.
- The crack tip opening displacement δ_t can be related to *J* by

$$\delta_t = f(\varepsilon_o, n) J / \sigma_o$$

Where *f* is a proportionality factor dependent on the yield strain ε_o and the strain hardening exponent *n*.

Critical *J* Integral (J_{IC})

- J_{IC} test:
  - 3-point bend
  - Compact tension (CT)

K_{IC} → materials resistance to crack extension

J_{IC} → toughness of the material near the outset of crack extension

- J_{IC} is numerically equal to G_{IC} values determined from valid plane-strain fracture toughness specimens for elastic condition,

$$J_{IC}(\text{plastic test}) = G_{IC} = \frac{K_{IC}{}^{2}}{E}(1-\nu^{2})(\text{elastic test})$$

- J is determined from a series of test specimens at different amounts of crack extensions *Δa*.
- Rice developed a simple model for the determination of the plastic component of J_{IC}. For the case of plate with a deep notch and subjected to bending

$$J_{pl} = \frac{2}{b}\int_0^{\delta_c} P d\delta_c$$

Where δ_c = displacement of sample containing a crack
P = load/unit thickness
b = unbroken ligament (W-a)

The equation reduces to $$J_{pl} = \frac{2A}{Bb}$$

Where B = specimen thickness
A = area under the load vs. displacement curve

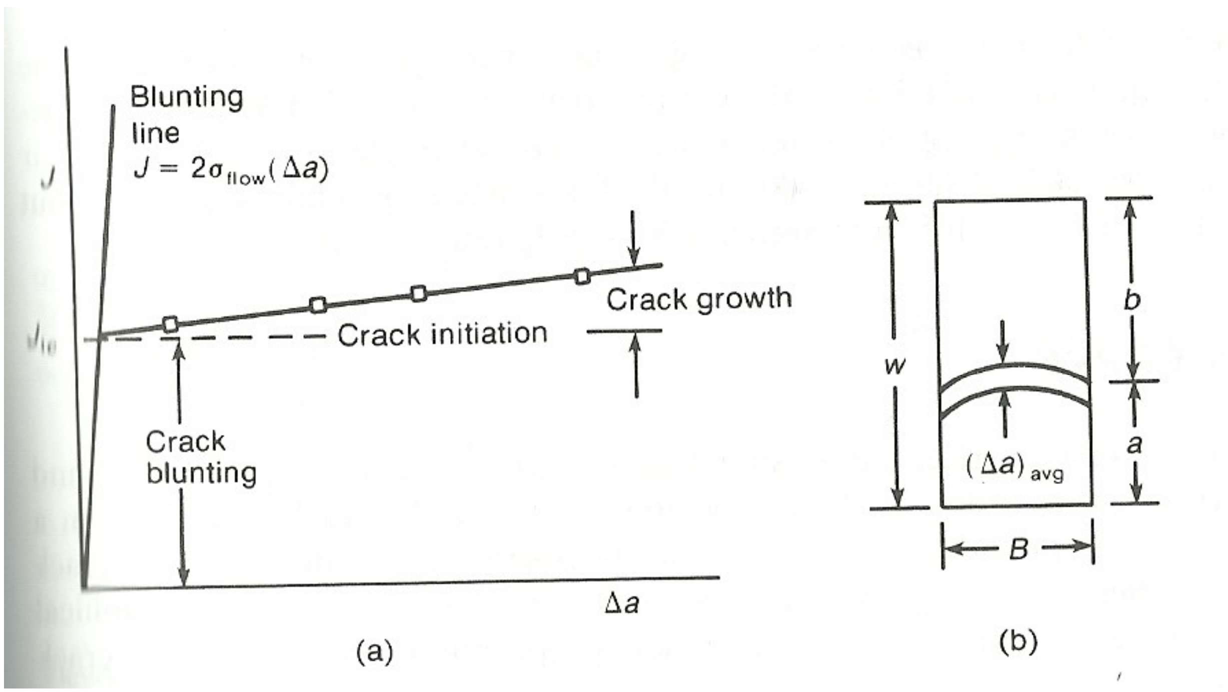
The above equ defines J for a 3-point bend specimen with a span/width ratio of 4

For compact tension specimen the equation is modified to account for the tensile loading component

$$J_{pl} = \frac{\eta A}{Bb} \text{ where } \eta = 2 + 0.522(b/W)$$

$$J = \frac{2A}{Bb} f\left(\frac{a_o}{b}\right)$$

a_o is initial crack length



J
Blunting
line
J = 2σflow(Δa)
Crack growth
J_Ic
Crack initiation
Crack
blunting
Δa
(a)
w
b
a
(Δa) avg
B
(b)

Specimens requirements for J_{IC} and K_{IC}

$$\text{for valid } K_{IC},\ a \text{ or } B \geq 2.5\left(\frac{K_{IC}}{\sigma_o}\right)^2$$

Whereas $J_Q = J_{\text{IC}}$ if $B,\ b > 25J_Q/\sigma_{o,}$ since

$$J_{IC} = \frac{K_{IC}^2}{E/(1-\nu^2)}$$

$$\frac{K_{IC}\text{ specimen size}}{J_{IC}\text{ specimen size}} = \frac{2.5\left(\frac{K_{IC}}{\sigma_o}\right)^2}{25\frac{K_{IC}^2}{E\sigma_o}} \geq 20$$