# Strain energy release rate
# (Fracture mechanics)

# *Orowan's modification for plastic deformation*

- Griffith performed experiments with glass and found that the difference in surface energy and fracture energy values is not great.
- However this is not true for metals and polymers, where the fracture energy is found to be several orders of magnitude higher than the surface energy.
- Orowan suggested that the Griffith equation can be written for brittle fracture in metals by inclusion of the plastic work, $\gamma_p$ required to extend the plastic work

$$\sigma_f = \left[\frac{2E(\gamma_s + \gamma_p)}{\pi a}\right]^{1/2} \approx \left(\frac{E\gamma_p}{a}\right)^{1/2}$$

Where $\gamma_p >> \gamma_s$

*$\gamma_p$ = $10^2$ to $10^3$ J/m$^2$ compared to $\gamma_s$ = 1 – 2 J/m$^2$.*

*a* = half crack length

$$\sigma_f = \left( \frac{2E\gamma_s}{\pi a} \left( \frac{\gamma_P}{\gamma_S} \right) \right)^{1/2} \qquad (1)$$

From cohesive theory $$\sigma_c = \left( \frac{E\gamma}{a_o} \right)^{1/2}$$

Maximum stress at crack tip, $\sigma_{\max} \approx 2\sigma_a \sqrt{a/\rho}$

The term $2\sqrt{a/\rho}$ is defined as the stress-conc. factor, $k_t$, and describes the effect of crack geometry on the local crack-tip stress level

At crack tip, $\sigma_c = \sigma_{max}$

$$\sigma_a = \frac{1}{2} \left( \frac{E\gamma_S}{a} \left( \frac{\rho}{a_o} \right) \right)^{1/2} = \left( \frac{2E\gamma_S}{\pi a} \left( \frac{\pi\rho}{8a_o} \right) \right)^{1/2} \qquad (2)$$

There is a similarity between equ 1 and equ 2 and it suggests a correlation between $\gamma_P/\gamma_S$ and $\pi\rho/8a_o$; the plastic deformation can be related to blunting of crack tip- $\rho$ will increase with $\gamma_P$.

Griffith relation is valid for sharp cracks with a tip radius in the range of $(8/\pi)a_o$ and when $\rho < (8/\pi)a_o$.

When $\rho > (8/\pi)a_o$. Equ 1 and 2 control the failure condition where plastic deformation is involved.

Irwin also considered the application of Griffith's relation to the materials with plastic deformation

Irwin chose to use the energy source term – the elastic energy per unit crack-length increment $\partial U/\partial a$, *i.e. G*.

Irwin showed that:

$$\sigma = \sqrt{\frac{EG}{\pi a}}$$

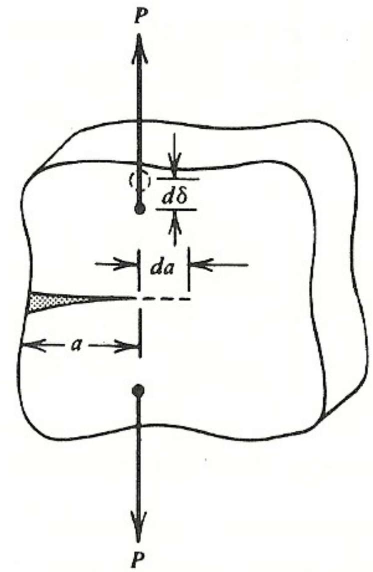
By comparing with Orowan's equ

$$G = 2(\gamma_S + \gamma_P)$$

At the point of instability, the elastic energy release rate, *G* reaches a critical value $G_c$, where fracture occurs. $G_c$ is called the fracture toughness of the material or critical strain energy release rate.

$G_c$ may be interpreted as a material parameter and can be measured in the Laboratory with sharply notched specimen

# Strain energy release rate



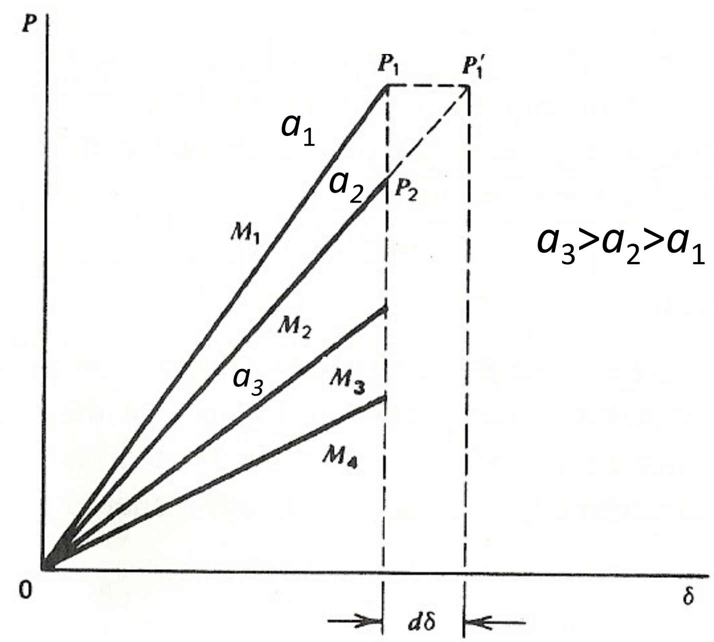

A single edge notch (sharpest possible notch) specimen is loaded axially.

Load (P) vs. displacement (δ) curves are determined for different length notches, where $P = M\delta$.
$M$ = stiffness of the material with crack length $a$.

The elastic strain energy is given by the area under the curve to a particular value of $P$ and $\delta$

$$U_o = \frac{1}{2} P\delta = \frac{P^2}{2M}$$

As shown in Fig, the increment of crack length *da* results in a drop in load from $P_1$ to $P_2$

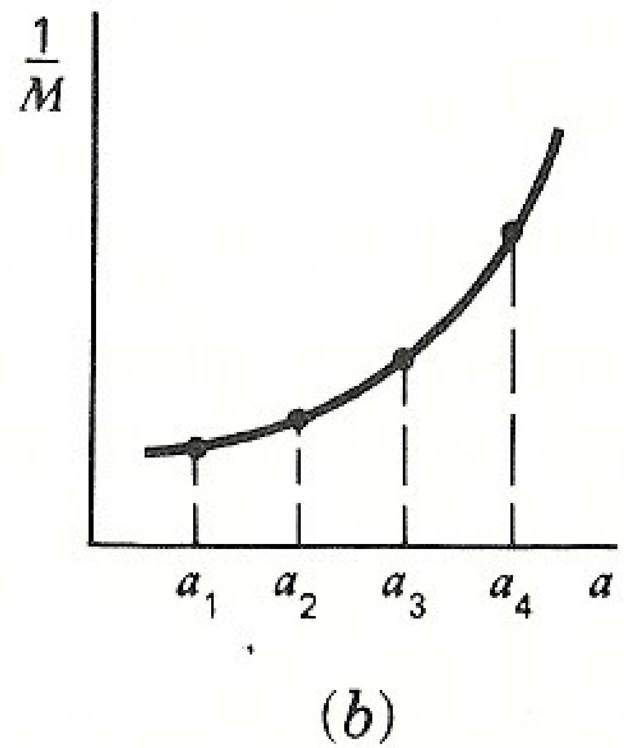



$a_3 > a_2 > a_1$

# Strain energy release rate

$$G_c = \frac{1}{2} P_{\max}^2 \frac{\partial(1/M)}{\partial a}$$

$1/M$ = compliance




Compliance dependence on crack length

The fracture toughness or critical strain energy release rate, is determined from the load, $P_{max}$, at which the crack propagates to fracture

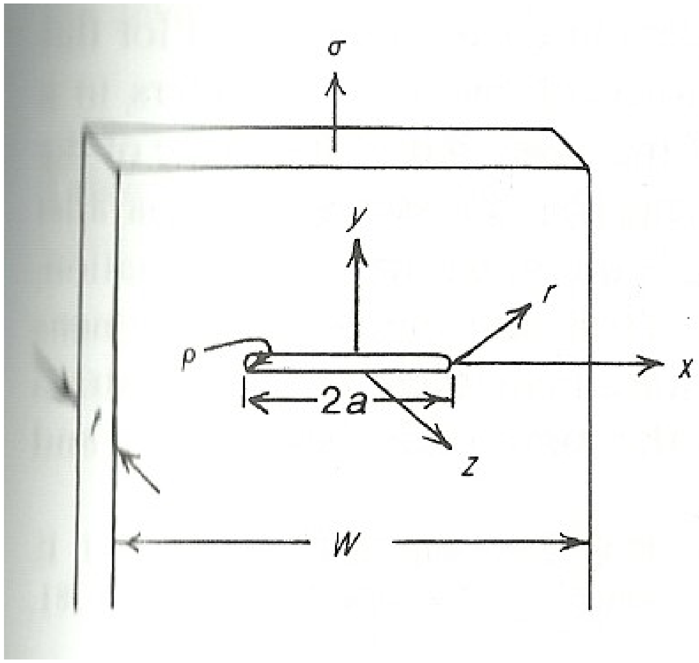
# Stress intensity factor

The stress distribution at the crack tip in a thin plate for an elastic solid is given by

$$\sigma_x = \sigma\left(\frac{a}{2r}\right)^{1/2}\left[\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\right]$$

$$\sigma_y = \sigma\left(\frac{a}{2r}\right)^{1/2}\left[\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\right]$$

$$\tau_{xy} = \sigma\left(\frac{a}{2r}\right)^{1/2}\left[\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right]$$




Where $\sigma$ = nominal stress = $P/wt$

The equs are valid for $a > r > \rho$

For an orientation directly ahead of the crack ($\theta = 0$)

$$\sigma_x = \sigma_y = \sigma\left(\frac{a}{2r}\right)^{1/2} \quad \text{and } \tau_{xy} = 0$$

Irwin showed that the local stresses near a crack depend on the product of the nominal stress, σ and the square root of the half-crack length.

He called this relationship the stress intensity factor, *K*

For a sharp elastic crack in an infinitely wide plate, *K* is given as

$$K = \sigma\sqrt{\pi a}$$

For a general case:

$$K = \alpha\sigma\sqrt{\pi a}$$

Where α depends on the specimen and crack geometry

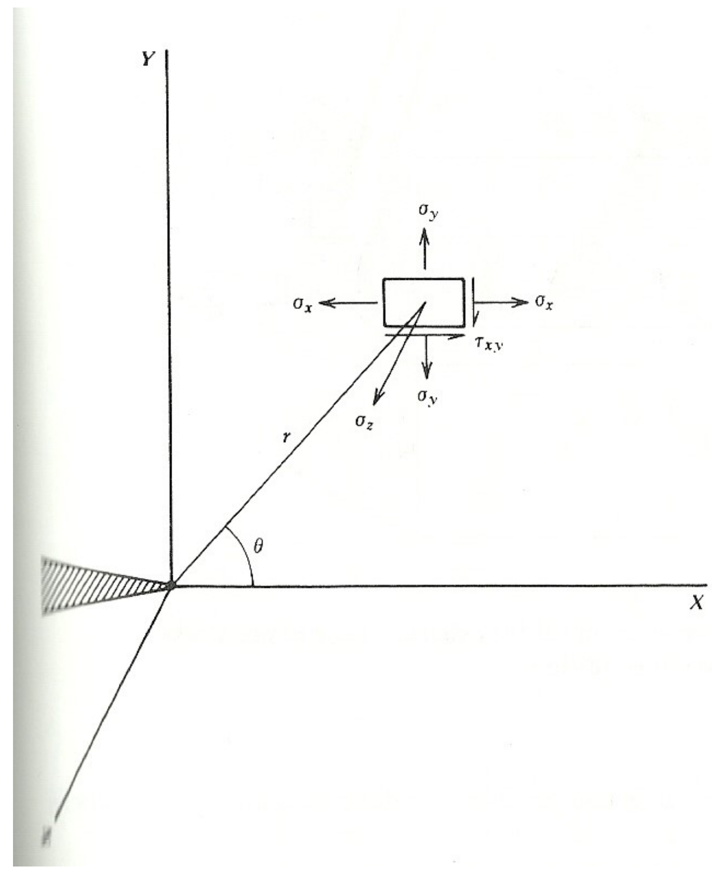
*K* expressed in $\mathrm{MPa}\sqrt{\mathrm{m}}$.

The stress field at the end of the crack can be written as

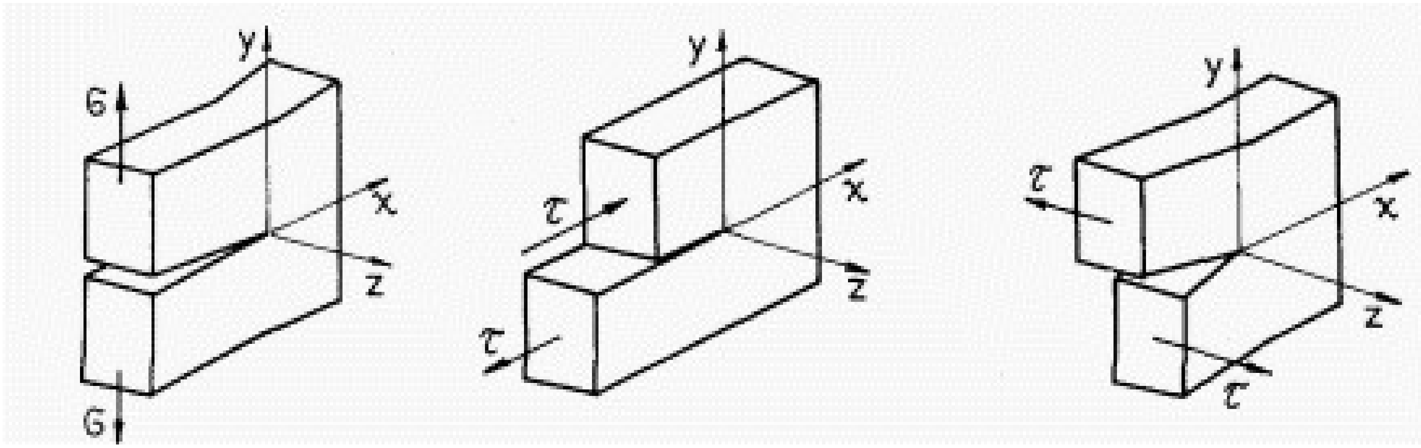
$$\sigma_y = \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_x = \frac{K}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}}\left(\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$



# Crack deformation modes



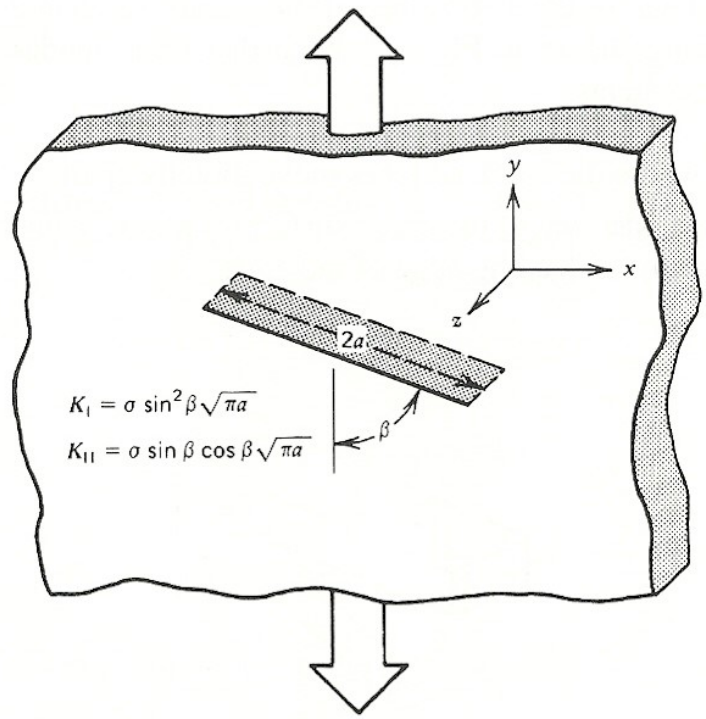

Mode I　　　　Mode II　　　　Mode III

**Mode I – *Opening mode*:** where the crack surfaces separate symmetrically with respect to the plane occupied by the crack prior to the deformation (results from normal stresses perpendicular to the crack plane);

**Mode II - *Sliding mode*:** where the crack surfaces glide over one another in opposite directions but in the same plane (results from in-plane shear); and

**Mode III - *Tearing mode*:** where the crack surfaces are displaced in the crack plane and parallel to the crack front (results from out-of-plane shear).

- Mode I loading has engineering importance. This is the usual mode for fracture-toughness tests and a critical value of stress intensity determined for this mode is designated as $K_{IC}$.
- Mixed Mode I-II involves axial loading of a crack (rotation about z-axis)
- Mode III is purely shear loading (notched round bar in torsion)
- When β > 60°, mode I contribution dominates.



## *Relation between strain energy and fracture toughness*

$$\sigma\sqrt{\pi a} = \sqrt{EG}$$

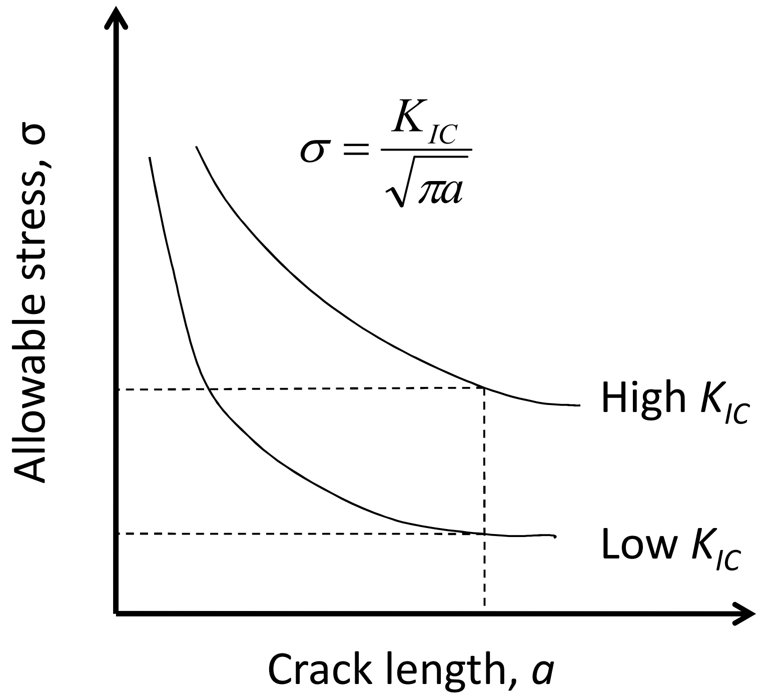
$$K = \sqrt{EG} \qquad \text{(plane stress)}$$

$$K = \sqrt{\frac{EG}{(1-\nu^2)}} \qquad \text{(plane strain)}$$

# Fracture toughness and design

- A properly determined value of $K_{IC}$ represents the fracture toughness of the material independent of crack length, geometry, or loading system.
- If the material is selected, $K_{IC}$ is fixed. Further if a large stable crack is allowed then the design stress is fixed and must be less than $K_{IC}$.
- If high strength and light weight are required, $K_{IC}$ is fixed because of the limited materials available and the stress level must be kept high because of the need to maximize payload. Therefore the allowable flaw size will be small.

| Material | Yield strength, MPa | Fracture toughness, $K_{IC}$, MPa m$^{1/2}$ |
|---|---|---|
| 4340 steel | 1470 | 46 |
| Maraging steel | 1730 | 90 |
| Ti-6Al-4V | 900 | 57 |
| 2024-T3 Al alloy | 385 | 26 |
| 7075-T6 Al alloy | 500 | 24 |




Relation between fracture toughness and allowable stress and crack size


Ref: 1. Mechanical Metallurgy, G.E. Dieter

2. Deformation and Fracture Mechanics of Engg Materials, Hertzberg