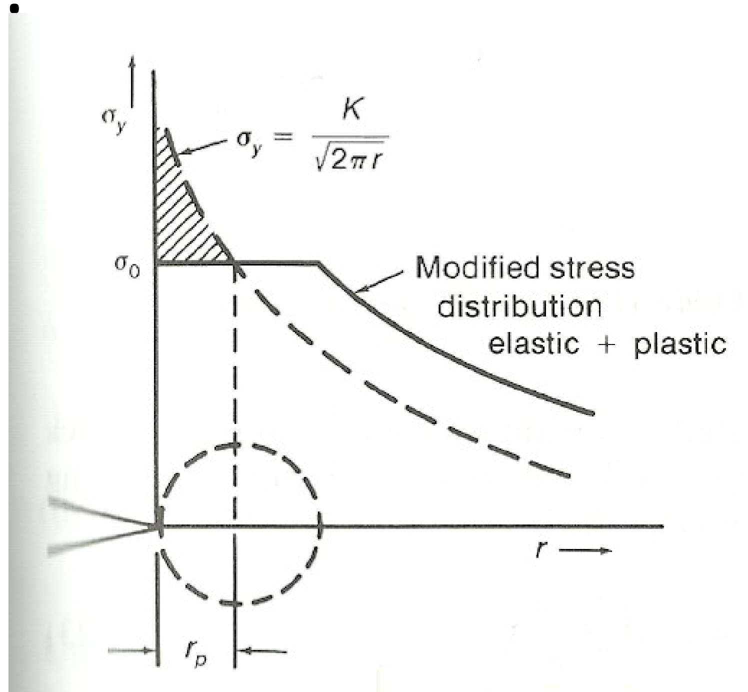
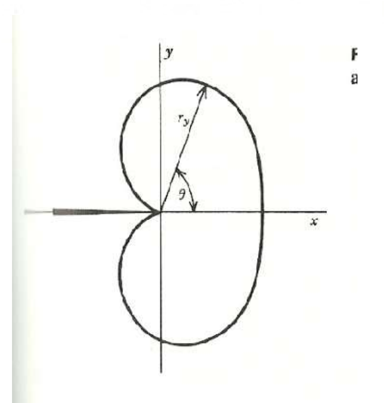
Plasticity correction

- Metals will yield when $\sigma = \sigma_o$ and a plastic zone will exist at the crack tip.
- Up to $r = r_p$, $\sigma_y > \sigma_o$, r_p = plastic zone size
- Consider the stresses existing directly ahead of the crack where $\theta=0$





At elastic-plastic boundary

$$\sigma_y = \frac{K}{\sqrt{2\pi r_p}} = \sigma_o$$

$$r_p = \frac{K^2}{2\pi\sigma_o^2} = \frac{\sigma^2 a}{2\sigma_o^2}$$

Irwin's correction

- Irwin proposed that the existence of a plastic zone makes the crack act as if it were longer than its physical size.
- As a result of crack-tip plasticity the displacements are larger and the stiffness is lower than for elastic situation.
- The effective crack length is the actual length plus the radius of the plastic zone.

$$a' = a + r_p$$

$$r_p \approx \frac{1}{2\pi}\frac{K^2}{\sigma_o^2} \qquad \text{(plane stress)}$$

where

$$r_p \approx \frac{1}{6\pi}\frac{K^2}{\sigma_o^2} \qquad \text{(plane strain)}$$

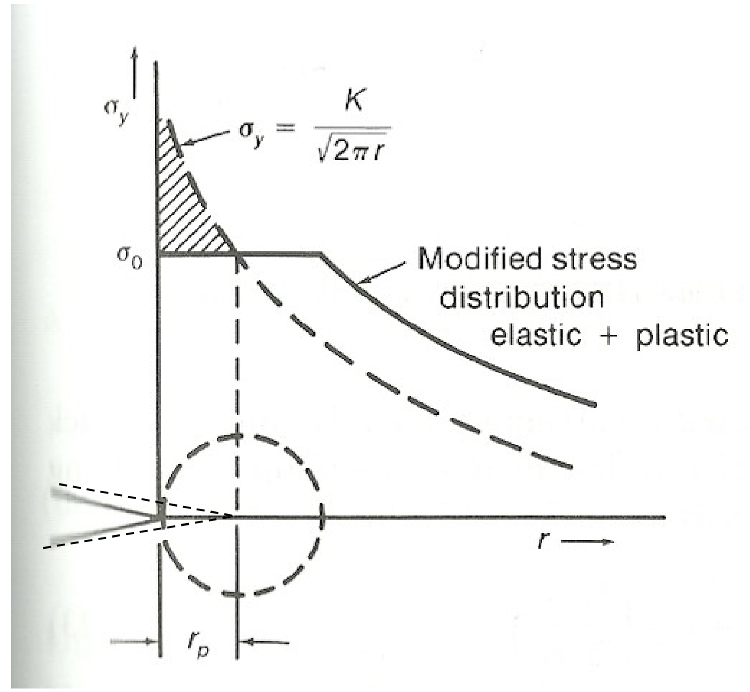
r_p for plane strain is smaller than plane stress, which is because the triaxial stress field suppresses the plastic deformation

$$K_{eff} = \sigma\sqrt{\pi a'}$$

K_{eff} = effective stress intensity factor

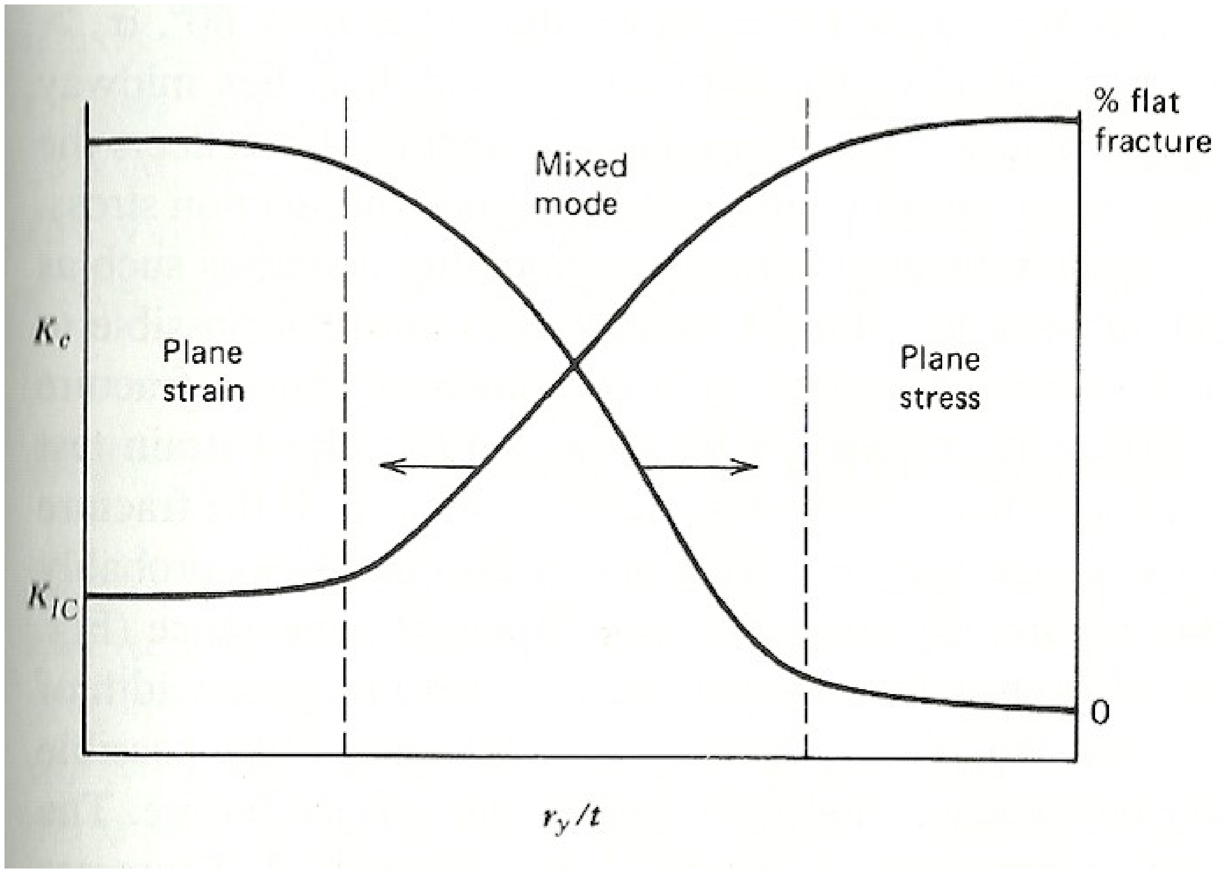
Irwin's correction to the fracture toughness of a center-notched tensile specimen

$$K = \sigma\sqrt{\pi a'}\left[\frac{w}{\pi a'}\tan\frac{\pi a'}{w} + \frac{1}{2w}\frac{K^2}{\sigma_o^2}\right]$$



Onset of plastic deformation at the crack-tip

Plane stress vs plane strain



Example

- A steel plate with a through thickness crack of length 2a = 20 mm is subjected to a stress of 400 Mpa normal to the crack. If the yield strength of the steel is 1500 Mpa, what is the plastic zone size and the stress intensity factor for the crack. Assume that the plate is infinitely wide.

Dugdale plastic strip model

Dugdale has proposed the crack tip plastic zone model for plane stress case.

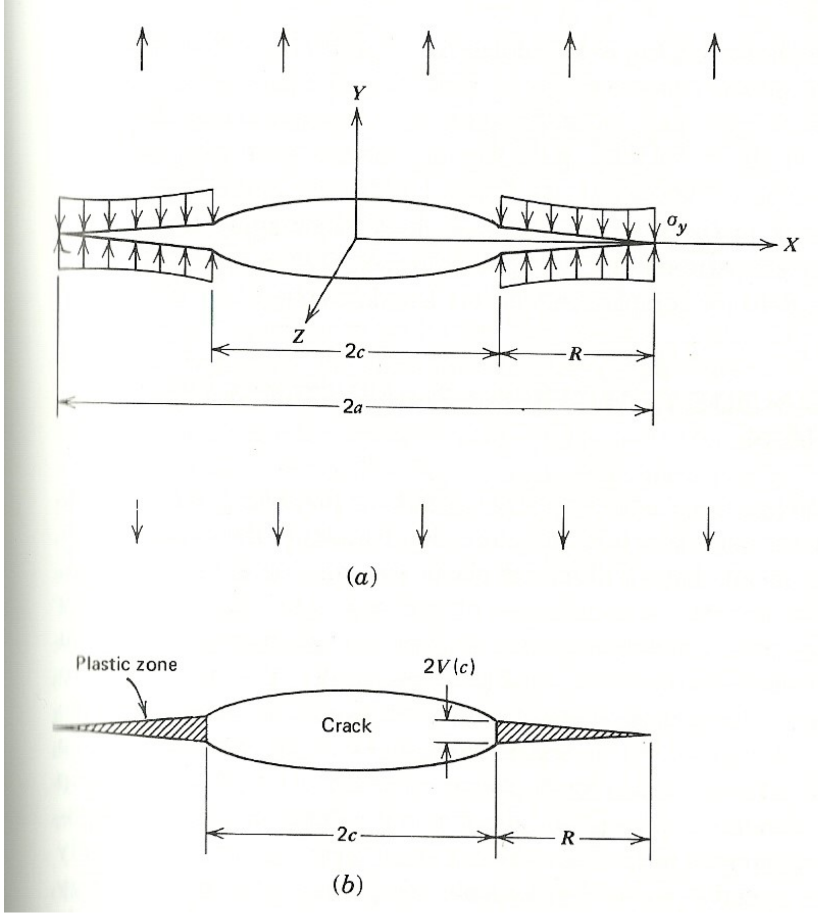
Dugdale considered the plastic regions to take a form of narrow strips extending a length R.

Assuming there is an elastic crack of length 2(c+R) and the region of length R is closed up by applying compressive stresses normal to the surface. Dugdale demonstrated that

$$c/a = \cos\left(\frac{\pi}{2}\frac{\sigma}{\sigma_o}\right)$$

Where a = c + R

$$R/c = \sec\left(\frac{\pi}{2}\frac{\sigma}{\sigma_o}\right) - 1$$



When the applied stresses $\sigma << \sigma_o$,

$$R = \frac{\pi^2\sigma^2 c}{8\sigma_o^2} = \frac{\pi K^2}{8\sigma_o^2}$$